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ON THE SYNTHESIS PROBLEM FOR AN INFINITE CYLINDER WITH AN AXIAL SLOT[+]

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Introduction

The problem of synthesizing a prescribed far-zone radiation pattern by means of a line source or by fields excited in an aperture in an infinite conducting plane has received extensive and sometimes elegant treatment in recent literature. The analogous problems for the exterior of an infinite conducting cylinder, which are equally important but of somewhat greater analytical difficulty, have suffered relative neglect. The present paper is an attempt to reduce this disparity. It seems, moreover, in regard to the plane case, that while the treatment given by Rhodes (1963), in terms of the eigenfunctions of the finite Fourier transform relating the aperture and far fields, may constitute an ultimate solution of the problem from a purely analytic standpoint, the consideration of certain more practical aspects might substantially modify the conclusions reached. The treatment here given the cylinder problem is accordingly based on a somewhat broader perspective, and the implications of some of the assumptions usually adopted at the outset in the literature on the plane case are brought into question.

Formulation of the Problem

The problem considered is that of an axial slot of width 2α $(0 < \alpha < \pi)$ on the surface of a conducting cylinder of infinite length and arbitrary radius a. It is assumed that there is no variation of the fields in the axial direction, so that the problem is essentially two-dimensional. In terms of the cylindrical coordinates (r, ϕ) the relationship between the tangential component of the electric field strength in the aperture and the radiation pattern can be expressed as

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$$P(\phi) = \frac{1}{2\pi} \int_{-\alpha}^{\alpha} K(\phi - \phi')\, A(\phi')\, d\phi' \qquad -\pi < \phi \leq \pi \tag{1}$$

wherein the quantities are defined for the two canonical cases as follows:

TM field	TE field
$E_z \underset{r\to\infty}{\sim} \sqrt{\frac{2}{\pi i}}\, \frac{e^{ikr}}{\sqrt{kr}}\, P(\phi)$	$E_\phi \underset{r\to\infty}{\sim} \sqrt{\frac{2i}{\pi}}\, \frac{e^{ikr}}{\sqrt{kr}}\, P(\phi)$
$E_z(a, \phi') = A(\phi')$	$E_\phi(a, \phi') = A(\phi')$
$\sum_{n=-\infty}^{\infty} \frac{e^{in\phi}}{i^n H_n^{(1)}(ka)} = K(\phi)$	$\sum_{n=-\infty}^{\infty} \frac{e^{in\phi}}{i^n H_n^{(1)'}(ka)} = K(\phi)$

Also, $k = \omega/c = 2\pi/\lambda$ is the wave number, and the time factor $e^{-i\omega t}$ is everywhere suppressed. $H_n^{(1)}$ and $H_n^{(1)'}$ are the Hankel functions of the first kind, of order n, and its derivative with respect to the argument.

Since the procedure is exactly the same for both cases, only the TM field is considered explicitly. Further, it is assumed that both functions $P(\phi)$ and $A(\phi')$ are square-integrable over their respective ranges of definition.

Equation (1) can be considered as an integral equation of the first kind for the unknown function $A(\phi')$, and due to the regularity of the kernel it will have in general no solution for arbitrary $P(\phi)$. It is, however, possible to approximate every $P(\phi) \in L^2(-\pi, \pi)$ arbitrarily closely in the mean-square sense by a pattern function corresponding to an $A(\phi) \in L^2(-\alpha, \alpha)$. A proof of this theorem is given, which, as in the plane case, assures the theoretical possibility of obtaining "supergain". As the mean-square deviation from the given pattern becomes smaller, however, the norm of the aperture function in general increases without limit, so that it is necessary to impose some constraint on this function in order to rule out forms which are obviously unrealistic.

A suitable constraint is to hold some "quality factor" for the system constant during the process of minimizing the error. Three possible alternatives are considered in this regard: 1) the analog to what is termed in the plane case the "supergain ratio" (Taylor, 1955); 2) the quality factor proposed by Chu (1948); and 3) that adopted by Collin and Rothschild (1964). All three have the property that a high value clearly implies an impractical design, but the converse is not necessarily true. Actually it is shown that an aperture field which arbitrarily small supergain ratio may still have an infinite value for either quality factor, and even if the values of these are moderate the realization of the design may be difficult if not impossible.

The analytical mechanism of the synthesis problem is formulated as follows. For a given radiation pattern $P_g(\phi)$ the optimum aperture distribution $A(\phi)$ and actual pattern $P(\phi)$ are those which achieve

$$\min_{A(\phi)\in L^2(-\alpha,\alpha)} \left\{ \frac{1}{\pi k Z_o} \int_{-\pi}^{\pi} \left| P_g(\phi) - P(\phi) \right|^2 d\phi + \mu \cdot 2\omega W_m \right\}.$$

Here W_m is the time-average magnetic energy stored in the evanescent field as defined by Collin and Rothschild and μ is a fixed parameter which can be considered as a weight factor imposed on the stored energy in relation to the mean-square deviation from the given pattern. Mathematically speaking μ is a Lagrange multiplier, and consequently the above condition gives the minimum of the mean-square deviation subject to the subsidiary condition $2\omega W_m \leqslant C$ const. Since the stored magnetic energy is always larger than the electric energy in the TM case, this condition is approximately equivalent to fixing the quality factor of Collin and Rothschild. The value of the Lagrange multiplier is contravariant with whatever quality factor is used, as a measure of the realizability of the optimum aperture distribution, and its import is thus the same. The advantage of fixing the multiplier rather than the quality factor during the minimization process is that since the latter depends implicitly on the former, there is no way of obtaining an explicit expression for the multiplier, and its determination would involve the iteration of the whole procedure with a sequence of trial values.

Quantitative Results

A substantial amount of numerical work has been undertaken with the aid of IBM computing facilities, according to the following general scheme:

The aperture function is first expressed in the form

$$A_N(\phi) = \sum_{m=0}^{N} \gamma_m \psi_m(\phi)$$

where $\{\psi_m(\phi)\}$ is a set of functions in $L^2(-\alpha,\alpha)$ so chosen that the quality factor considered is finite. (This requires that the aperture function, and consequently the functions $\psi_m(\phi)$, satisfy the Meixner edge condition at the boundaries of the slot.) The aperture function corresponding to the optimum pattern is thus theoretically obtainable as $\lim_{N\to\infty} A_N(\phi)$. For every value of N, the coefficients $\gamma_o, \gamma_1, \ldots \gamma_N$ are given explicitly as the solution of a system of linear equations which results from the minimizing conditions. The corresponding matrices are inverted for a reasonable range of values of the parameters μ, α, ka, and N. Once the inverse matrices have been determined, the optimum pattern and aperture functions, the mean-square deviation, and the quality factor for any prescribed pattern with the chosen set of parameter values are obtainable through a relatively simple computation.

In this way considerable information is obtained, at modest expense, on the complex interdependence of the various factors involved in the antenna design. Such questions as, for example, the price of using a narrow slot, in terms of resulting quality factor for a given deviation from a given pattern, or the relative importance of the quality factor constraint in synthesizing different types of pattern, are easily answered. The prescribed patterns considered include both omnidirectional and narrow-beam types. Certain cases are included for comparison in which no constraint is imposed other than the requirement that the aperture function be represented by a Fourier series with finite number of terms. Some inference is drawn on the effect of the quality factor constraint on the limit of the mean-square deviation as the number of terms in the aperture field expansion increases without bound.

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